HW 4 Due October 25th

1. Is the following a vector space (provide reason)? If yes, provide the dimension and find a basis.
   a) All vectors in $\mathbb{R}^3$ with $v_1 - v_2 + 2v_3 = 0$
   b) All vectors in $\mathbb{R}^3$ with $v_1 \geq v_2$

2. Find the limit state of the Markov process modeled by the following matrices:
   a) $\begin{bmatrix} 0.2 & 0.5 \\ 0.8 & 0.5 \end{bmatrix}$
   b) $\begin{bmatrix} 0.6 & 0.1 & 0.2 \\ 0.4 & 0.1 & 0.4 \\ 0 & 0.8 & 0.4 \end{bmatrix}$

3. Are the eigenvalues of $A + B$ sums of the eigenvalues of $A$ and $B$?

4. Prove one of the following:
   a. Eigenvectors of a symmetric matrix corresponding to different eigenvalues are orthogonal.
   b. Inverse of a skew-symmetric matrix is skew-symmetric.

5. What kind of conic section is given by the quadratic forms below:
   a. $3x_1^2 + 8x_1x_2 - 3x_2^2 = 10$
   b. $9x_1^2 + 6x_1x_2 + x_2^2 = 10$

6. Find an eigenbasis (a basis of eigenvectors) and diagonalize. Show the details of your work.

$$A = \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}$$

$$B = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & -4 \end{bmatrix}$$